Thus, the dependence of Nu on Re in transverse flow around the lateral surface of a cylinder is a broken line with straight-line segments when $Re \geq 7 \cdot 10^3$; this must be taken into account in designing thermoanemometric sensors.

NOTATION

Re, Nu, Reynolds and Nusselt numbers; Nu_0, C, n, constants defined in the text.

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EXCITATION OF THERMOACOUSTIC OSCILLATIONS IN A HEATED CHANNEL

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It is shown that one of the main causes of thermoacoustic oscillations in heated channels is associated with positive work done by bubbles in a sound wave. The sign of the work depends on the characteristics of the bubbles: their size, velocity, and heat and mass transfer with the surrounding liquid.

Thermoacoustic oscillations [1-4] can arise in a heated channel in which the relatively cool surface boiling region occupies a significant fraction of the heated length of the channel. The amplitudes of these oscillations can reach values of the order of the average pressure in the channel in the case when the heated region is relatively short and the underheated, relatively cool region is significant. Thermoacoustic oscillations appear in the form of several different modes (usually three).

Thermoacoustic oscillations can lead to undesirable phenomena: a disturbance of the operating conditions of the device, a lowering of the critical heat loads, and a premature collapse of the channel as a result of fatigue heat loads. The excitation of thermoacoustic oscillations in heated vapor-generating channels at subcritical pressures has been studied mainly experimentally [1-4], and not very extensively. A number of suggestions have been put forth on the mechanism of the excitation of thermoacoustic oscillations, however all of them are mainly qualitative in nature. There is currently no rigorous quantitative treatment available describing the conditions for excitation of thermoacoustic oscillations in surface boiling.

In the present paper we extend the approach developed in [2, 5]. In this approach the excitation of thermoacoustic oscillations depends on the work done by the bubbles in a sound wave over a period of oscillation. The work A done by the bubble over a period of oscillation T in the acoustic pressure field is the intrinsic contribution of the bubble to the excitation of thermoacoustic oscillations. If $A > 0$, then over a period of oscillation the bubble does positive work on the surrounding liquid and this leads to a "build-up" of the oscillation. If $A < 0$ the bubble stabilizes the process. The perturbations of the pressure and volume of the bubble can be written in the form

$$\delta P = a_P \sin \omega t, \tag{1}$$

$$\delta V = a_V \sin (\omega t + \beta). \tag{2}$$

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Then the resulting work done by the bubble over a period T of the sound wave can be obtained using (1) and (2):

$$A=\int_0^T \delta P \frac{d\delta V}{dt} dt = -\pi a_P a_V \sin\beta. \quad (3)$$

According to [6], the quantity β in (3) is a phase-frequency characteristic of the bubble as a linearized nonlinear system responding to the acoustic field. The work A is positive if $\sin\beta < 0$, i.e.

$$-\pi<\beta<0. \quad (4)$$

The condition (4) will be satisfied if the imaginary part of the amplitude-phase characteristic of the bubble is negative [6]:

$$\operatorname{Im}\frac{\delta\tilde{V}}{\delta\tilde{P}}(j\omega)<0. \quad (5)$$

Formally putting $a_P a_V = \pi^{-1}$, $a_P = 1$, the reduced work is

$$A=-\operatorname{Im}\frac{\delta\tilde{V}}{\delta\tilde{P}}(j\omega). \quad (6)$$

Experiments [2, 8] in heated channels and also calculations show that thermoacoustic oscillations exist when the true volume vapor content near the channel outlet is $\phi \approx 0.1$-0.6. For such values of ϕ the number of bubbles moving in the bulk of the flow is much larger than the number of bubbles near the walls of the channel. It follows from the calculations of [7] that the vapor content ϕ "created" by the bubbles near the walls is usually less than 0.01-0.04. Therefore we can assume that most of the bubbles in the channel are moving in the bulk of the flow. We will assume that the contribution of the bubbles near the walls to the excitation of thermoacoustic oscillations is second order because of their small relative number.

In order to use (6), we need information on the distribution of the bubble parameters (radius, slip velocity, interfacial interactions, and so on) with respect to length along the channel.

The necessary distributions of the bubble parameters can be obtained by using the polydispersed nonequilibrium model of a two-phase fluid containing bubbles worked out in [7]. According to this model, each group of bubbles with the same conditions of nucleation and growth is described by a set of equations of the form

$$F_c+F_m+V\frac{\partial P}{\partial z}=0, \quad (7)$$

$$\frac{\partial\rho''V}{\partial t}+W_b\frac{\partial\rho''V}{\partial z}=q_{LR}(W_b,\ W_L,\ \ldots)\,Sr^{-1}, \quad (8)$$

where

$$F_c=\frac{C_D S\rho_L}{2}\Delta W\,|\Delta W|;\ F_m=\frac{\rho_L V(1-\varphi)}{2}\left[\frac{\partial\Delta W}{\partial t}+W_b\frac{\partial W_b}{\partial z}-\right.$$
$$\left.-W_L\frac{\partial W_L}{\partial z}+\frac{1}{V}\Delta W\frac{\partial V}{\partial t}+\frac{W_b}{V}\Delta W\frac{\partial V}{\partial z}\right].$$

Expressions for $C_D(\Delta W,\ \phi,\ R)$ and $q_{LR}(W_b,\ W_L,\ \Delta T_U,\ldots)$ are given in [7, 9].

For oscillations about the unperturbed state of the system the relevant variables are written in the form [6]

$$b(t)=\bar{b}(t)+\delta\tilde{b}\exp(j\omega t). \quad (9)$$

Here $\bar{b}(t) \gg \delta\tilde{b}$. We will also assume that the velocity of the medium and the bubbles is much smaller than the speed of sound of the fluid. Then we can assume that $|\partial b/\partial t| \gg |W(\partial b/\partial z)|$. Linearizing (7) and (8) and using (9), we obtain the following relation for the perturbation of the volume of the bubble:

$$\delta\tilde{V}=\mu_1\delta\tilde{P}+\mu_2\delta\tilde{W}_L+\mu_3\frac{d\delta\tilde{P}}{dz}, \quad (10)$$

where μ_1, μ_2, μ_3 are known functions of the parameters of the bubble W_b, V, q_{LR}, S, and the transform parameter $s = j\omega$. Here the parameters W_b, V, q_{LR}, S are functions of z' and z, where z' is the coordinate of the nucleation of the bubble and z is the current coordinate of the bubble. We now use (10) to find the transfer function (6). We first eliminate δW_L from (10) using a simplified equation of motion of the mixture in the form

$$\rho_{mix}\frac{\partial W_{mix}}{\partial t} = -\frac{\partial P}{\partial z} \tag{11}$$

and put $W_L \approx W_{mix}$, $\rho_L = \rho_{mix}$ for small ϕ. We then obtain (11) in terms of the perturbations

$$\delta\tilde{W}_L = -\frac{1}{s\rho_L}\frac{d\delta\tilde{P}}{dz}. \tag{12}$$

Substituting (2) into (10), we have

$$\partial\tilde{V} = \mu_1(z',\ z,\ s)\,\delta\tilde{P} + \left[\mu_3(z',\ z,\ s) - \frac{\mu_2(z',\ z,\ s)}{s\rho_L}\right]\frac{d\delta\tilde{P}}{dz}. \tag{13}$$

It is known experimentally [2] that near the boundaries of the oscillation excitation region the distribution of the amplitude with length along the channel is nearly harmonic, i.e., we can write

$$\delta P(z,\ t) = \delta\bar{P}(t)\sin\left(\frac{2\pi n}{H}z\right) \tag{14}$$

or

$$\delta\tilde{P}(z,\ s) = \delta\tilde{\bar{P}}(s)\sin\left(\frac{2\pi n}{H}z\right), \tag{15}$$

where n = 1, 2, 3,... is the mode of oscillation. Using (15), we obtain from (10) the amplitude-frequency characteristic of the bubble as a function of the position of its nucleation and its current position in the flow:

$$\frac{\delta\tilde{V}}{\delta\tilde{\bar{P}}} = \Pi(s) = \mu_1(z',\ z,\ s)\sin\left(\frac{2\pi n}{H}z\right) + \left[\mu_3(z',\ z,\ s) - \frac{\mu_2(z',\ z,\ s)}{s\rho_L}\right]\frac{2\pi n}{H}\cos\left(\frac{2\pi n}{H}z\right). \tag{16}$$

Substituting (16) into (6), we obtain the required expression for the work done by the bubble in the flow. Here μ_1, μ_2, μ_3 depend on the position of nucleation and the position of the bubble in the flow, the history of the variation of the bubble parameters W_b and V in the field of the variable flow parameters (T_L, W_L, P, and so on). As noted above, this information can be obtained theoretically with the help of the model of [7]. Because the equations expressing the conservation laws for the bubbles are written in Eulerian coordinates, the degradation of a given bubble in the Lagrangian representation does not affect the final results. In any cross section z lying above the cross section where the bubble began to separate there exist bubbles with the appropriate parameters V(z', z), W_b(z', z), and so on as a result of continuous "reproduction" of bubbles in nucleation cross sections z'. Therefore the results obtained from (16) do not depend on the relation between the lifetime of a bubble (in Lagrangian coordinates) and the period of oscillation of the sound wave.

Figure 1 shows a set of graphs of the reduced work of the bubbles $\sum_i A_i N_i$ in the bulk of the flow as a function of length along the channel, where i corresponds to groups of bubbles having the same growth history and parameters in a given cross section of the flow. The starting points of the curves are the coordinates of the separation of the bubbles according to the model of [7]. The curves are numbered in order of increasing heat load q supplied to the channel for the same values of the other parameters of the flow [T_L(z = 0); P(z = 0); ρW]. In the calculations on which Fig. 1 is based, we used the data of our earlier experiments on the regions of stability of underheated boiling fluids [8]. According to the experiments, for the conditions of Fig. 1, the heat load q_{low} at which thermoacoustic oscillations begin to occur and the value of the heat load q_{high} at which the thermoacoustic oscillations cease when q is increased are q_{low} = 0.80 MW/m² and q_{high} = 0.92 MW/m². Curve

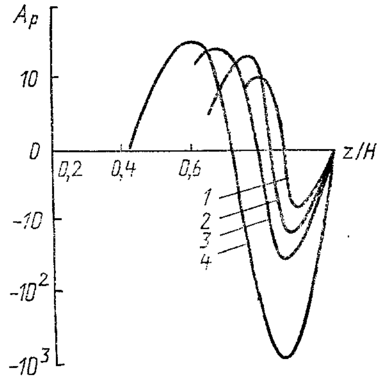

Fig. 1. Work of the bubbles $A_p = \sum_i A_i N_i / (a_p)^2_{max}$, Pa^{-1} as a function of length along the channel (ρW = 1000 kg·m^{-2}·sec^{-1}, T_{up} = 423 K; P = 12.0 MPa): 1) q = 0.78 MW/m^2; 2) q = q_{low} = 0.8 MW/m^2; 3) q = 0.82 MW/m^2; 4) q > q_{high} = 0.92 MW/m^2.

1 corresponds to the stable case (q = 0.78 MW/m^2). In this case the total work done by all bubbles in the channel is negative (the total area included between the curve and the z/H axis). As q increases, the negative part of the work increases more slowly than the positive part. When $q > q_{low}$ the total work is positive and thermoacoustic oscillations can arise and be maintained by the bubbles. Upon further increase of q ($q > q_{high}$) the total work again becomes negative (curve 4). Therefore the behavior of the curves $\sum_i A_i N_i(z/H)$ qualitatively determines the nature of the motion in the channel (stable or unstable against thermoacoustic oscillations). Similar results were obtained for the other experimental data in the region of the parameters studied in [8]: P ≤ 16.0 MPa; ρW = 500-2000 kg/(m^2·sec); q ≤ 2.5 MW/m^2. It also follows from the calculations based on (6) and (16) that bubbles near the channel outlet stabilize the oscillation process. This is because of the decrease in ΔT_U as well as the increase in the volume $\bar{V}$ of the bubbles. In the case of uniform heating of the entire channel the main contribution to destabilization comes from bubbles in the middle and in the third quarter of the channel. To study the effect of bubble size, in the final set of calculations (for the case of motion on the lower stability boundary) we assumed constant values of the bubble radius R. It follows from the calculations that as the bubble radius is decreased and the other parameters are held constant, the "positive" part of the total work increases and the process is stabilized. Similar calculations were carried out for fixed values of ΔT_U and $\Delta W = W_b - W_L$. It was found that an increase in ΔT_U and ΔW also destabilizes the process.

In the experiments basically only the first harmonic was excited. Therefore we put n = 1 in the calculations. Calculations with n = 2 and 3 showed that the contributions of these modes to the total work was small and decreased with increasing n.

It follows that the excitation of thermoacoustic oscillations in two-phase fluids containing bubbles is determined mainly by the behavior of bubbles moving in the bulk of the flow due to the acoustic pressure field. The condition (4), which was obtained by considering the work done by a bubble in the acoustic pressure field, is related to the well-known Rayleigh criterion [10]: thermoacoustic oscillations are excited when the phase shift between the pressure oscillations $\delta P = a_P \sin \omega t$ and the oscillations of the mass supply (heat supply) per unit time $\delta G = a_G \sin(\omega t + \beta)$ satisfies the condition

$$|\beta| < \frac{\pi}{2}. \tag{17}$$

We represent (16) in the form

$$\delta \tilde{V} = \Pi(j\omega)\,\delta\tilde{P} = [\mathrm{Re}\,\Pi(j\omega) + j\,\mathrm{Im}\,\Pi(j\omega)]\,\delta\tilde{P}. \tag{18}$$

Since $\rho'' \ll \rho'$ the change of the mass of vapor per unit volume per unit time as a result of a conversion of liquid into vapor (or vice versa) can be ascribed to "external" mass supply for a compressible two-phase medium. In this case the perturbation of the rate of mass supply of vapor per unit volume is approximately

$$\delta G \approx N_{av} \frac{\partial}{\partial t} \left(\frac{d\rho''}{dP} V_{av} \delta P + \rho'' \delta V_{av} \right), \tag{19}$$

where for simplicity V_{av} is the average bubble volume in a unit volume with concentration N_{av}. Then with the help of (18) and (19), we obtain

$$\frac{\delta \tilde{G}}{\delta \tilde{P}} = \rho'' N_{av} \quad \operatorname{Im} \Pi(j\omega) + j\omega N_{av} \left[\frac{d\rho''}{dP} V + \rho'' \operatorname{Re}(j\omega) \right]. \tag{20}$$

It follows from (20) that the phase shift between δG and δP will correspond to the condition (17) when $\operatorname{Im} \Pi(j\omega) < 0$. Hence the conditions for the excitation of thermoacoustic oscillations in two-phase fluids obtained from the Rayleigh criterion and from the work done by a bubble in the acoustic field are equivalent, for the assumptions considered above.

Therefore a bubble moving in a two-phase fluid participates in the excitation of thermoacoustic oscillations most strongly when the perturbation of the rate of change of the mass of vapor in the bubble and the perturbation of the pressure have the same phase ($\beta = 0$).

The mechanism of excitation of thermoacoustic oscillations can be summarized as follows. In an acoustically isolated channel there exists noise, which can be represented as a sum of standing waves of different frequencies (modes). These standing pressure waves act on bubbles, resulting in a perturbation of the mass of vapor in them. If the perturbation of the mass of vapor is in phase with the perturbation of the pressure, then the bubbles maintain and amplify the pressure oscillation. Suppose, for example, that a condensing bubble is located at an antinode of the pressure wave and moves without slipping. In this case as the pressure increases, the surface area of the bubble (i.e., its surface of condensation) decreases, which leads to an increase in the mass of vapor in the bubble in comparison with the unperturbed state. This growth in the mass of vapor in the bubble causes a further increase in the pressure perturbation. When the pressure is decreased the opposite situation occurs: the mass of vapor in the bubble decreases in comparison with the unperturbed state. Hence an increase (decrease) in the pressure in the sound wave leads to a change in the size of the bubble which promotes a further increase (decrease) in the perturbation, i.e., the oscillation is amplified.

This excitation mechanism of thermoacoustic oscillations is approximate, since in a real system there are other effects superimposed on this mechanism which complicate the process. For example, the gradient of the pressure perturbation acting on a bubble depends on its position. Hence the effect of the pressure perturbation on the parameters of the bubble and on the contribution of the bubble to the thermoacoustic oscillations will depend on the position of the bubble.

Therefore the growth of thermoacoustic oscillations in two-phase fluids containing bubbles depends on how the pressure perturbation from the sound wave affects the bubble parameters in each cross section of the channel. The perturbation of the bubble can either promote the growth of the perturbation or hinder it. In the framework of our approach this coupling is taken into account by (16).

NOTATION

P, pressure; $\omega = 2\pi T^{-1}$; T, period of oscillation; t, time; $V = 4\pi R^3/3$, volume of a bubble; a_p, a_V, positive real functions; β, phase shift; Re, Im, real and imaginary parts; $j^2 = \sqrt{-1}$; $M_V = \rho''_V$; ρ'', ρ_L, densities of vapor and liquid; r, latent heat of vaporization; T_L, temperature of the liquid; C''_S, temperature of the vapor on the saturation line; λ_L, thermal conductivity of the liquid; Π, transfer function; a_L, thermal diffusivity of the liquid; σ, surface tension; ΔT_U, underheating of the liquid; Ja, Jacobi number; q_{LR}, specific heat flux between the bubbles and the liquid; ν_L, kinematic viscosity of the liquid; W_b, W_L, velocity of the bubbles and velocity of the liquid averaged over a cross section; S, surface area of a bubble; $\Delta W = W_b - W_L$; $s = j\omega$; ϕ, true volume vapor content; H, height of the channel; N, number of bubbles per unit volume; q, specific external heat flux; ρW, mass velocity.

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THEORETICAL MODEL OF NONEQUILIBRIUM OF EXTRACTION OF A GAS DISSOLVED IN A FLUID DURING PRESSURE FLUCTUATIONS IN A FLOW

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An engineering model and a method of computing nonequilibrium extraction of a gas dissolved in a fluid during pressure fluctuations in a dispersely-nucleate gas-liquid flow that differs from a known flow by taking account of the turbulent nature of the relative phase motion and a more accurate determination of the gas bubble radius in an equivalent flow. The results of computing the magnitude of the additional gas-extraction during pressure fluctuations in a pipeline are compared with experimental data obtained earlier in water and carbon dioxide that verifies the reliability of the developed theoretical model.

INTRODUCTION

Low-frequency pressure fluctuations due either to vibration loaded hydraulic systems or to cavitation self-scillations in their elements occur in their hydraulic systems during the operation of the majority of power plants.

As experimental investigations [1] showed, the pressure fluctuations in a gas-liquid flow cause additional gas extraction from the fluid with respect to the stationary case, that can result in a cavitation collapse of the supply pump operation. The reason causing additional gas extraction is the periodic change in the surface area of the phase separation in the flow. Under the action of pressure fluctuations on a gas-fluid flow the process of gas extraction alternates with the dissolution process. However, the mass transfer surface area will be greater during pressure diminution than during magnification. Consequently, more gas is extracted during one fluctuation period than is dissolved.

A theoretical analysis of this phenomenon, called rectified gas diffusion, was first performed in the Harvard Acoustic Research Laboratory and its results are represented in [2, 3]. Later more modern theoretical models [4-6] were developed, however, they are all based on a spherically-symmetric formulation of the gas diffusion problem without taking account of the relative motion of the gas bubbles and the fluid. At the same time a substantial relative motion of the gas and liquid phases occurs at the same time in power plant mainline supply systems. E. V. Vengerskii [7], who developed an engineering model and method during pressure fluctuations in a gas-liquid flow, performed a theoretical analysis of the rectified gas diffusion in this case.

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